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SUBSTANTIATION OF A STOCHASTIC MODEL OF
RHYTHMIC PHENOMENA

by

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FOREIGN AEROSPACE SCIENCE & TECHNOLOGY CENTER

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Block	Italic	Transliteration	Block	Italic	Transliteration
А а	*А а*	A, a	Р р	*Р р*	R, r
Б б	*Б б*	B, b	С с	*С с*	S, s
В в	*В в*	V, v	Т т	*Т т*	T, t
Г г	*Г г*	G, g	У у	*У у*	U, u
Д д	*Д д*	D, d	Ф ф	*Ф ф*	F, f
Е е	*Е е*	Ye, ye; E, e*	Х х	*Х х*	Kh, kh
Ж ж	*Ж ж*	Zh, zh	Ц ц	*Ц ц*	Ts, ts
З з	*З з*	Z, z	Ч ч	*Ч ч*	Ch, ch
И и	*И и*	I, i	Ш ш	*Ш ш*	Sh, sh
Й й	*Й й*	Y, y	Щ щ	*Щ щ*	Shch, shch
К к	*К к*	K, k	Ъ ъ	*Ъ ъ*	"
Л л	*Л л*	L, l	Ы ы	*Ы ы*	Y, y
М м	*М м*	M, m	Ь ь	*Ь ь*	'
Н н	*Н н*	N, n	Э э	*Э э*	E, e
О о	*О о*	O, o	Ю ю	*Ю ю*	Yu, yu
П п	*П п*	P, p	Я я	*Я я*	Ya, ya

*ye initially, after vowels, and after ъ, ь; e elsewhere.
When written as ё in Russian, transliterate as yё or ё.

RUSSIAN AND ENGLISH TRIGONOMETRIC FUNCTIONS

Russian	English	Russian	English	Russian	English
sin	sin	sh	sinh	arc sh	$\sinh^{-1}$
cos	cos	ch	cosh	arc ch	$\cosh^{-1}$
tg	tan	th	tanh	arc th	$\tanh^{-1}$
ctg	cot	cth	coth	arc cth	$\coth^{-1}$
sec	sec	sch	sech	arc sch	sech^{-1}
cosec	csc	csch	csch	arc csch	csch^{-1}

Russian	English
rot	curl
lg	log

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SUBSTANTIATION OF A STOCHASTIC MODEL OF RHYTHMIC PHENOMENA

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Submitted 25 Jun 70

In a number of areas of science and technology it is necessary to deal with rhythmic processes, such as the parasitic modulation of shot fluctuations in radio-electronic equipment [8,9], changes in the levels of stability of oscillatory parameters of biological systems [12], modulation of an acoustic field in a layer of water due to heaving of its surface [7, 13], change in meteorological parameters under the action of tidal forces [14], etc. Many investigators [1, 11, 12] note the advisability of using methods of the theory of stationary random processes for analyzing such essentially nonstationary phenomena as biological, etc. rhythms, and stress the necessity of developing statistical methods, based directly on the properties of this type of nonstationary processes and having an explicit interpretation. The frequently used additive model - the determinate periodic function $f(t)$ on a background of stationary noise $\nu(t) - f(t) + \nu(t)$, analyzable by methods of the theory of processes with stationary pulsations, does not describe the main property of rhythmic phenomena - change in the parameters of the process, analogous to pulsations (of modulation). A multiplicative model of the type of modulation - $\xi(t) f(t)$, where $\xi(t)$ - stationary process, although it describes this feature it is not general. Therefore the need arose for developing a general probability model of rhythmic phenomena and on this base to develop effective methods for analyzing them. Although in the literature [11] the important role of a study of periodic nonstationary random processes is noted in connection with an analysis of rhythmic phenomena, this thesis is in no way substantiated, and the theory of this class of processes is found in the initial phase of development, to say nothing of its applications. In work [8] only the process of a type of modulation is studied. Taking this into consideration, the author together with K.S. Voychishin made an attempt at developing and substantiating a general model of rhythmic phenomena. The idea of creating such a model as a base for developing methods of analyzing them, the use for this purpose of periodically correlated (PK) random processes (SP)*, and the proof of the necessary theorems belong to the author, and for substantiation of the model the results from processing the materials of observations of the bioactivity of loaches and meteorological elements which were obtained by K.S. Voychishin were used considerably.

* Footnote. Here, just as in work [5], we will use the term "periodically correlated" in the sense of "periodically nonstationary of the second order" (or in a wide sense, if an analogy with stationary processes is used). This class of processes is sufficient for the theory being developed. Periodic nonstationary in a narrow sense is defined, as usual, through the periodic nature of n-dimensional distributions of probabilities [11]. [End of footnote]

For creating a meaningful theory, in contrast to mathematical, it is not sufficient just to select axioms, it is also necessary that they represent significant sides of the phenomenon for which the theory is being constructed. Therefore in work [2] on the basis of an analysis of tracings of rhythmic phenomena it is postulated that if a phenomenon takes place in an unchanged mode, then it is reasonable to accept the following as its significant characteristics: a) noise-like nature, i.e., the fundamental impossibility of their representation in the form of a complex determinate function; b) harmonizability - separability into simple harmonic components; c) nonstationary state of the type of modulation, characterized by a periodic change in dispersion, and this means in covariation.

From these postulates it follows [6] that in the correlation theory a periodically correlated random process (PKSP) should be a general model of a rhythmic phenomenon.

The purpose of this article, just like that of article [6], developing the ideas of work [5] and supplementing work [2], is a further study of the structural properties of PKSP as a model of rhythms, but in contrast to article [6] the mathematical expectation of PKSP, generally speaking, is considered different from zero.

<u>Definition 1.</u> We will call the random process (SP) centered if its mathematical expectation is equal to zero.

Then from a presentation of PKSP through stationary components (SK) [6]

$$\xi(t) = \sum_{k=-\infty}^{\infty} \xi_k(t) e^{ik\frac{2\pi}{T}t} \qquad (1)$$

and the uniqueness of the expansion into a Fourier series follows the validity of the theorem.

Theorem 1. A PKSP is centered when, and only when, its stationary components are centered.

Theorem 2. If the SK PKSP is uncorrelated and centered, with the exception maybe of zero, then it degenerates into into a stationary SP, if the SK are only uncorrelated, then into SP with stationary pulsations and a periodic mean.

Actually, from correlation [5]

$$b_{\xi}(t,u)=\sum_{k=-\infty}^{\infty}B_k(u)e^{ik\frac{2\pi}{T}t}=\sum_{k=-\infty}^{\infty}\sum_{l=-\infty}^{\infty}r_{l,l-k}(u)e^{i\frac{2\pi}{T}(kt-lu)} \qquad (2)$$

(where $r_{k,j}(\cdot)$ - mutual covariation of k- and j-th SK under the condition of noncorrelation of SK $r_{k,j}(u)=r_k(u)\,\delta_{j,k}$, from which $r_{l,l-k}(u)=r_l(u)\,\delta_{k,0}$) we obtain

$$b_{\xi}(t,u)=\sum_{l=-\infty}^{\infty}r_l(u)e^{il\frac{2\pi}{T}u}=B_0(u),$$

where $B_0(\cdot)$ - covariation of a stationary approximation to PKSP [5], i.e., SP $\xi(t)$ is stationary relative to covariation. And for the mathematical expectation from formula (1) we find

$$m_{\xi}(t)=\sum_{k=-\infty}^{\infty}m_k e^{ik\frac{2\pi}{T}t},$$

where $m_k=E\,\xi_k(t)$ - in a general case (for all $m_k\neq 0$) a periodic function with a period of correlation.

Theorem 3. The non-correlatedness of odd SK PKSP with even and paired together is sufficient so that its covariation would possess half a period.

Actually, under the conditions of the theorem

$$r_{j,k}(u)=\begin{cases}0, & \text{if } (j\vee k) \text{ are odd}\\ 1, & \text{if } (j\wedge k) \text{ are even,}\end{cases}$$

and then in equality (2) $B_k(u)=\sum_{n=-\infty}^{\infty}r_{n,n-k}(u)=\sum_{q=-\infty}^{\infty}r_{2q,2q-k}(u)$ is identically equal to zero in the case of odd k.

Corollary 1. Under the conditions of the theorem a PKSP possesses a half a period if its odd SK are centered.

Corollary 2. The absence of SK is sufficient so that the PKSP would possess half a period.

Corollary 3. Centered SK, non-correlated pairwise and with all the remainder, can be related to zero.

When all the SK are linear transforms of one of them (for example, this will happen when the PKSP is obtained by conversion of a stationary SP by a system with periodically changing parameters):

$$\xi_k(t) = A_k\xi(t) = \int_{-\infty}^{\infty} e^{it\lambda}\psi_k(\lambda)\,dZ(\lambda),$$

where $\psi_k(\cdot)$ - frequency characteristic of the corresponding invariant filter A_k, then

$$\xi(t) = \int_{-\infty}^{\infty} e^{it\mu} \sum_{k=-\infty}^{\infty} \psi_k\left(\mu - k\frac{2\pi}{T}\right) dZ\left(\mu - k\frac{2\pi}{T}\right)$$

and the correlation components (KK)

$$B_k(u) = \int_{-\infty}^{\infty} e^{iu\mu} \sum_{l=-\infty}^{\infty} \psi_l\left(\mu - l\frac{2\pi}{T}\right)\varphi_{l-k}\left(\mu - l\frac{2\pi}{T}\right) dS\left(\mu - l\frac{2\pi}{T}\right).$$

and the covariations of components

$$r_{kj}(u) = \int_{-\infty}^{\infty} e^{iu\lambda}\psi_k(\lambda)\,\overline{\psi_j(\lambda)}\,dS(\lambda).$$

From here it is evident that the components will be non-correlated if $\psi_k(\lambda)\,\overline{\psi_j(\lambda)}=0.$

In the particular case under consideration it is directly evident that a situation is possible when $B_k(0)=0$ and $B_k(u)\neq 0$ in the case of $u\neq 0$. For this it is sufficient to assume that the j-th SK is the k-th Hilbert transform , i.e., $\psi_k(\lambda)\equiv 1$, and $\psi_j(\lambda)=-i\,\mathrm{sgn}\,\lambda$, since

$$r_{k,j}(u) = 2\int_0^{\infty} \sin\lambda\, dS_k(\lambda).$$

Example. Assume

$$\eta(t) = \xi(t)e^{i\Lambda t} + \check{\xi}(t)e^{-i\Lambda t},$$

where $\vee$ - denotes the conversion to the Hilbert transform, then taking $r_{\hat{\xi}}(u)=r_{\xi}(u)$ and $r_{\xi,\check{\xi}}(u)=-r_{\check{\xi},\xi}(-u)$ into account we obtain

$b_\eta(t,u) = 2r_\xi(u)\cos\Lambda u + B_2(u)e^{i2\Lambda t}$, where $B_2(u) = r_{\xi\hat{\xi}}(u)e^{i\Lambda u} - r_{\xi\hat{\xi}}(-u)e^{-i\Lambda u}$, from here $B_2=0$. The dispersion of this process $\sigma_\eta^2(t) = 2r_\xi(0) = 2\sigma_\xi^2 = \text{const}$. If the process $\xi(t)$ is noncentered and $E\xi(t)=m$, then $m_\eta(t)=2m\cos\Lambda t$.

In the case when the converter is a differential operator with periodic coefficients $\{a^l(t), l=\overline{1,N}\}$, then $\psi_k(\lambda) = \sum_{l=0}^{N} a_k^l(i\lambda)^l$, where a_k^l - the k-th coefficient of the Fourier function $a^l(\cdot)$. When only the actual process in the steady-state mode are considered, then odd l are absent and the period of correlatedness of the process will be equal to half of the period of change of the parameters of the converter. Then, having designated the product of the series

$$P_{kj}(\lambda) = \sum_{l=-\infty}^{\infty} a_l^k(i\lambda)^l \sum_{n=-\infty}^{\infty} a_n^j(i\lambda)^n = \sum_{m=-\infty}^{\infty}\left[\sum_{s=-\infty}^{\infty} a_{m-s}^k a_s^j\right](i\lambda) \ ,$$

for the covariation SK we find

$$r_{kj}(u) = \int_{-\infty}^{\infty} e^{iu\lambda}P_{kj}(\lambda)\,dS(\lambda).$$

If we designate $s^+(\lambda) = \frac{1}{2}[s(\lambda) + s(-\lambda)]$ an even part of spectral density and take into account the evenness of functions $a_l^k(\cdot)$, then

$$r_{2k,2j}(0) = 2\int_0^{\infty} P_{2k,2j}(\lambda^2)\,s^+(\lambda)\,d\lambda,$$

since the integral with an odd part is equal to zero. From here the initial value of KK according to formula (2) $B_{2k}(0) = \int_0^{\infty} Q_{2k}(\lambda^2)s^+(\lambda)\,d\lambda$, where $Q_{2k}(\lambda^2) = 2\sum_{l=0}^{N} P_{2l,2(l-k)}(\lambda^2)$. For example, for operators of the second and fourth orders

$$Q_{2k}(\lambda^2) = \lambda^4 \sum_{l=-\infty}^{\infty} a_{2l} a_{2(l-k)}.$$

$$Q_{2k}(\lambda^2) = \lambda^8 \sum_{l=-\infty}^{\infty} a_{2l}^4 a_{2(l-k)}^4 - \lambda^6 \sum_{l=-\infty}^{\infty} [a_{2l}^4 a_{2(l-k)}^2 - a_{2(l-k)}^4 a_{2l}^2] + \lambda^2 \sum_{l=-\infty}^{\infty} a_{2l}^2 a_{2(l-k)}^2.$$

It follows from this that the values of KK in zero only in exceptional cases will be zeros, i.e., for steady-state processes, representing transforms of stationary processes with the help of differential operators with periodic coefficients, as a rule, the dispersion and mathematical expectation (if the processes are not centered) possess a precise period, equal to the period of correlatedness and are half of the period of change of the operator coefficients. Thus the PKSP, the result of a stationary disturbance of a system with periodically changing parameters, will possess a period which is half the period of change of its parameters.

The null covariation component describes the distribution of energy by harmonics which make up the PKSP [6], and therefore its role in the characteristics of the process is not at all different from the role of other KK, which is also evident from the following theorems.

Theorem 4. In order that the zero KK be identically equal to zero it is necessary and sufficient that it be equal to zero in zero.

This follows directly from the fact that $B_0(\cdot)$ is a covariation of a stationary approximation to PKSP, and the covariation of the stationary process satisfies the condition $r_\xi(u) \leq r_\xi(0) = \sigma_\xi^2$, which in this case gives the inequality $|B_0(u)| \leq B_0(u)$.

From formula (2) we find that $B_0(0) = \sum_{l=-\infty}^{\infty} r_l(0)$, and since all $r_l(0) = \sigma_l^2 > 0$ as dispersions of stationary processes, then the equality $B_0(0)=0$ is possible only in the case when all $\sigma_l=0$. But since from this same formula (2) $|B_k(u)| < \sum_{l=-\infty}^{\infty} |r_{l,l-k}(u)|$ and the mutual covariation of stationary processes satisfies the inequality $|r_{lk}(u)|^2 < r_l(0) r_k(0)$, then from here follows the conclusion, which, in view of its importance, we will formulate as a lemma.

Lemma 1. If $B_0(0)=0$, then all KK are identically equal to zero.

Actually, since when $\bar{B}_0(0)=0$ pulsations are absent, the process degenerates into a determinate function. Thus the following theorem takes place.

Theorem 5. Equality to zero of the value in zero of a zero KK is necessary and sufficient so that the PKSP would degenerate into a determinate function (equal to its average).

The sufficiency of the condition follows from formula (8), lemma 1 and the definition of dispersion $\sigma_\xi^2(t) = b_\xi(t,0)$. And the necessity is evident from the fact that the dispersion of pulsations of a determinate function is equal to zero. Then from the property of covariation $|b_\xi(t,u)| \leq \sigma_\xi(t+u)\,\sigma_\xi(t)$ it follows that $|b_\xi(t,u)| \leq 0$, and since $|B_k(u)| \leq \frac{1}{T}\int_0^T |b_\xi(t,u)|\,dt$, then $B_k(u)=0$ in the case of all k and u.

<u>Definition 2.</u> As normalization of SP we will name, in an analogy with normalization of random variables, the transition from process $\xi(t)$ to process

$$\tilde{\xi}(t) = \frac{\overset{\circ}{\xi}(t)}{\sigma_\xi(t)} = \frac{\xi(t) - m_\xi(t)}{\sigma_\xi(t)}. \tag{3}$$

Its covariation

$$b_{\tilde{\xi}}(t,u) = \frac{b_\xi(t,u)}{\sigma_\xi(t+u)\,\sigma_\xi(t)}, \tag{4}$$

and the average and dispersion, naturally, are defined as $m_{\tilde{\xi}}(t)=0, \sigma_{\tilde{\xi}}(t)=1$ in the case of all t. Therefore the unique characteristic of rhythm of a normalized PKSP can only be covariation. Since formulas (3) and (4) have meaning, when $\sigma_\xi(t) \neq 0$ for all t, then from formula (2) it follows that the function $\sigma_\xi(t+u)\,\sigma_\xi(t) = \sum_{m=-\infty}^{\infty} e^{im\frac{2\pi}{T}t} \sum_{l=-\infty}^{\infty} B_{m-l}(0)\,B_l(0)\,e^{il\frac{2\pi}{T}u}$ is periodic with respect to t with a period T and is non-vanishing.

Then based on the Wiener theorem [3] the function $[\sigma_\xi(t+u)\,\sigma_\xi(t)]^{-1}$ is expanded into an absolutely convergent trigonometric series of the same order.

Since the product of the periodic functions of one period is a periodic function of the same period, then this proves the validity of the theorem.

Theorem 6. The periodic correlatedness of an SP is invariant with respect to its normalization.

In particular, the covariation of a normalized PKSP, possessing a constant dispersion (evidently $\sigma_{\xi}^{2}(t)=B_0(0)$), will be $b_{\tilde{\xi}}(t,u)=\sum_{k=-\infty}^{\infty}\frac{B_k(u)}{B_0(0)}e^{ik\frac{2\pi}{T}t}$. It is advisable that normalization of processes by applied in the case of a comparative study of the rhythm of diverse-scale physical and other parameters.

Assume now that $\eta(t)$ - a stationary SP with the covariation $R_{\eta}(\cdot)$, and $f(t)=\int_{-\infty}^{\infty}e^{it\lambda}dF(\lambda)$, where $F(\cdot)$ - certain complex measure of limited variation, then the process

$$\xi(t)=\eta(t)f(t) \tag{5}$$

is harmonizable [10] and its function of covariation

$$b_{\xi}(t,u)=R_{\eta}(u)f(t+u)\overline{f(t)}, \tag{6}$$

and dispersion

$$\sigma_{\xi}^{2}(t)=\sigma_{\eta}^{2}|f(t)|^{2}. \tag{7}$$

Then the normalized function of covariation

$$b_{\xi}(t,u)=\varrho_{\eta}(u)e^{i[\arg f(t+u)-\arg f(t)]},$$

where $\varrho_{\eta}(u)=\frac{R_{\eta}(u)}{\sigma_{\eta}}$.

Definition 3. A process of the type (5) we will call a process of the type of simple modulation.

If the function $f(\cdot)$ is periodic and $\{c_k\}$ - its Fourier coefficients, then the KK of the process $\xi(t)$ will be

$$B_k(u) = R_\eta(u) \sum_{l=-\infty}^{\infty} \bar{c}_{l-k} c_l e^{il\frac{2\pi}{T}u}.$$

The following theorems stem from the previous formulas.

Theorem 7. A process of the type of simple modulation with an actual periodic modulating function is a PKSP which has been normalized to stationary.

Actually under the conditions of the theorem $b_\xi(t, u) = \rho_\eta(u)$.

Theorem 8. A symmetry of the third kind of a modulating function of an SP of the type of simple modulation is a necessary and sufficient condition of the fact that the period of its dispersion would be half the period of the correlated state.

The condition of symmetry of the third kind is fulfillment of the equality $f\left(t+\frac{T}{2}\right) = -f(t)$ in the case of all t, therefore the function $|f(\cdot)|$ possesses a period $\frac{T}{2}$ and confirmation of the theorem follows from formula (7).

Theorem 9. Simple modulation is necessary and sufficient for the conversion of white noise into nonstationary white noise, and if the modulating function is periodic, then this is PK noise [6].

The necessity follows from the definition of nonstationary white noise and the fact that the negative function is a square of the modulus of the function $g(t)=|f(t)|^2 \geq 0$, and the sufficiency is found from formula (6) and the property of the δ-function: $\delta(x-a) f(x) = \delta(x-a) f(a)$.

From the analysis made of the PKSP it is evident that the precise period of dispersion depends on the values in the zero of the KK, and the precise period of the average - on the centrality of the SK. Since in a general case these properties are independent, then the period of dispersion and the period of the average can be used for estimating the precise period of correlatedness of the PKSP. In this case the dispersion is more significant than the average, which frequently is considered equal to zero [4]. The most significant characteristic is the function of covariation, describing the correlatedness of the harmonic components of the PKSP, and its period is determined by the KK.

In particular, for a normalized PKSP the function of covariation is the unique characteristic of rhythm.

It is evident from formula (1) that when all SK are proportional to one of them, which can be considered as zero, i.e., when $\xi_K(t)=\alpha_K \xi_0(t)$, where $\{\alpha_K\}$ - complex numbers, then the PKSP turns into a process of the type of simple modulation

$$\xi(t) = \xi_0(t) \sum_{k=-\infty}^{\infty} \alpha_k e^{ik\frac{2\pi}{T}t} = \xi_0(t) f(t).$$

If all the SK are non-correlated and non-centered, then the PKSP turns into an additive model - a periodic determinate function on a background of stationary noise:

$$\xi(t) = \sum_{k=-\infty}^{\infty} m_k e^{ik\frac{2\pi}{T}t} + \eta(t),$$

where $\eta(t)$ - stationary SP with a null average and the covariation

$$R_\eta(u) = \sum_{k=-\infty}^{\infty} r_k(u) e^{ik\frac{2\pi}{T}u}.$$

It is evident from the considerations presented that the idea of isolation of the determinate component of the rhythmic phenomenon and the study of the stationary residual turns out to be inconsistent, since only in an additive model can it have direct meaning. In this case the stationary residual cannot describe the rhythmic nature, and in other models the residual is not stationary.

The theorems proved above jointly with the theorems in works [5,6] serve as substantiation of the model of rhythmic phenomenon as PKSP and reveal a series of its significant properties, and also show that neither additive nor multiplicative models can be considered as the most general for rhythmic phenomena, biological in particular, which both by isolation of mathematical expectation and by normalization are not reduced to stationary processes.

REFERENCES

1. Бартлетт М. С. Введение в теорию случайных процессов. ИЛ, М., 1958.
2. Войчишин К. С., Драган Я. П.— В кн.: Отбор и передача информации, 29. «Наукова думка», К., 1971.
3. Гельфанд И. М., Райков Д. А., Шилов Г. Е. Коммутативные нормированные кольца. Физматгиз, М., 1960.
4. Гладышев Е. Г.— Теория вероятностей и ее применения, 1963, VIII, 2.
5. Драган Я. П.— В кн.: Отбор и передача информации, 22. «Наукова думка», К., 1969.
6. Драган Я. П.— В кн.: Отбор и передача информации, 30. «Наукова думка», К., 1971.
7. Драган Я. П., Михайловский В. Н., Яворский И. Н.— В кн.: Отбор и передача информации, 28. «Наукова думка», К., 1971.
8. Козел С. М.— В кн.: Труды МФТИ, IV, М., 1959.
9. Малахов А. Н. Флуктуации в автоколебательных системах. «Наука», М., 1968.
10. Прохоров Ю. В., Розанов Ю. А. Теория вероятностей. «Наука», М., 1967.
11. Романенко А. Ф., Сергеев Г. А. Вопросы прикладного анализа случайных процессов. «Сов. радио», М., 1968.
12. Сольбергер А.— В кн.: Биологические часы. «Мир», М., 1964.
13. Толстой И., Елей К. С. Акустика океана. «Мир», М., 1969.
14. Brier G. W.— Monthly weather Keview, 1965, 93, 2.

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